Abstract

The Nevalinna Pick problem is a complex analysis problem, named after the Finnish mathematician Rolf Nevanlinna and the Austrian mathematician Georg Pick. The question is the following: given initial data consisting of n points $\lambda_1, \ldots, \lambda_n$ in the complex unit disc D and target data consisting of n points $z_1, \ldots, z_n$ in D, can we find an analytic function from the disk into itself that interpolates the data? The problem was independently solved independently by Pick in 1916 and by Nevalinna in 1919. Over the years, further generalizations have been studied and solved and lots of applications in several fields have been found. In particular, among reproducing kernel Hilbert spaces (RKHS), the ones which satisfy an analog of the property above are called Pick spaces. There are many well-known examples of such spaces. Some of their properties fall within general Pick theory, while other important ones have to be proved case by case. From this comes the idea of focusing on the study of a really simple and concrete space, that will be introduced in Chapter 3. All the results and proofs about this space which are presented here, are new. Chapter 1 of this thesis provides an overview of the general theory: we begin by introducing the fundamental concepts of RKHS and some key examples. Furthermore, we will present all the notions needed in the other chapters of this work. The main reference for this chapter is [AgMcC2002]. Chapter 2 introduces the tree model: it is a relatively simple example of RKHS with complete Pick property constructed on a tree. The main reference for this chapter is [Ro2019]. As said before, the main results of this thesis concern the space presented in Chapter 3. It comes as a generalization of the tree example, taking the tree given by the integers and passing to the continuous case. So, in Chapter 3, we present this complete Pick space of functions defined on the real line. This space is examined in details: many proofs of the Pick property are presented; interpolating sequences, multipliers, Carleson measures are characterized; the Corona problem is solved. Lastly, in Chapter 3 we study invariant subspaces in the Pick space under examination. Finally, Chapter 4 provides additional information about the previous space, showing some connections with Brownian motion and the Volterra integral operator. Some of the results from Chapter 3 where proved in collaboration with Nikolaos Chalmoukis (Universit`a di Milano-Bicocca), my co-advisor. Some questions were posed by Michael Hartz (Universit¨at des Saarlandes), my other co-advisor.